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Monte Carlo Computation of Rarefied Supersonic Flow into a Pitot Probe

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Pitot probes are commonly used to make measurements in high-speed gas flows in wind tunnels and vacuum chambers. In supersonic flow, a shock wave forms in front of the probe. In the continuum flow regime, the Rayleigh pitot probe equation is used to describe nonisentropic pressure losses across this shock. Under the rarefied flow conditions typical of supersonic plumes produced by spacecraft propulsion systems, the continuum theory is inaccurate. Hence, pressure corrections are needed to make use of pitot probe measurements in these flows. Currently, there is no general analytical theory describing this behavior. In the present study, the direct simulation Monte Carlo method is used to simulate rarefied, supersonic flow of nitrogen into a representative pitot probe geometry in order to predict these pressure corrections. Numerical results are compared with experimental data and are found to show good agreement qualitatively and quantitatively. Pressure corrections computed at very low probe Reynolds numbers are found to be consistent with free molecular limits calculated using collisionless gas theory.

Nomenclature

L/D	= length-to-diameter ratio of probe
M	= Mach number
P_{PG}	= measured pitot probe pressure
P_{T1}	= total pressure of external flow
P_{T2}	= total pressure behind ideal shock
Re_M	= modified probe Reynolds number
Re_P	= probe Reynolds number
T_T	= total temperature of external flow
T_W	= temperature of pitot probe wall
γ	= ratio of specific heats
(ρ_1/ρ_2)	= density ratio across ideal shock

Introduction

SATELLITES require propulsion systems for attitude control and station keeping. Low-thrust electric thrusters are being developed to fulfill this need. An important issue involved in the use of control thrusters is the effect of exhaust plume impingement on the spacecraft. Interaction of the plume with the spacecraft can cause unwanted heating or electrical charging of surfaces. Solar arrays and electronic devices may lose effectiveness under long-term exposure to rocket exhaust gases. Impact of the plume on the spacecraft produces disturbances torques. Each of these effects reduces the useful life span of a spacecraft. Correct characterization of the rocket flowfield is critical to the understanding of these impingement effects.

Numerical techniques are under development for the accurate prediction of these plume flows. For calibration of the numerical methods, reliable and accurate measurement devices and techniques are needed to provide useful and correct experimental results. One common device used in experimental investigations of plume flows is the pitot probe. In the continuum regime, the Rayleigh pitot tube equation[1] accurately describes conditions behind the shock that forms in front of the probe. The use of pitot probes in the plumes of low-thrust rockets is complicated by nonequilibrium effects that arise due to the rarefied, supersonic nature of the flow. Because of rarefaction effects, there are insufficient collisions to fully compress the shock that forms about the probe. As a result, the pressure loss across the shock is less than that predicted by continuum fluid mechanics. This effect must be corrected for when making pressure measurements using pitot probes under rarefied conditions.

Except in the free molecular limit, there is no comprehensive theory describing the interaction of a rarefied gas flow with a pitot probe. Although results from a number of experimental investigations are available, only a limited range of gas temperatures and probe Reynolds numbers have been considered.[2,3] Recent investigations of plume flows utilizing pitot probes have used pressure corrections extrapolated from the available experimental data.[4,5]

The goal of the current study is to investigate numerically the interaction between a rarefied, supersonic gas flow and a pitot probe. The direct simulation Monte Carlo (DSMC) technique is used to model the flow. The DSMC method employs a large number of model particles to simulate the behavior of the actual gas molecules. Because of the fundamental nature of the simulation method, DSMC is capable of capturing nonequilibrium effects that are of primary importance in the rarefied probe flow. The technique has been applied successfully to model nozzle and plume flows expanding from small rockets.[4–7]

Supersonic flow into a representative pitot probe geometry is considered at various conditions of freestream total temperature, total pressure, and Mach number. Nitrogen is used as the test gas due to both the availability of experimental data for diatomic gases and the use of nitrogen in previous plume studies.[4,5] Comparisons are made between pressure correction factors calculated using DSMC and those available from experimental measurements. The approach of these correction factors to the expected continuum and free molecular limiting values is investigated. Comparison is made with theoretical predictions for free molecular flow conditions. The dependence of pressure correction trends on freestream Mach number in the extremely rarefied regime is considered.

Pitot Probe Theory and Experiment

Pitot probes are used to measure the local total pressure of a gas flow. When used to investigate supersonic flows, a shock forms in front of the probe and the pressure that is measured is the stagnation pressure behind the shock. Under continuum conditions, this measured pressure can be related to the total pressure of the flow using the Rayleigh shock relations that can be represented by

$$P_{PG}/P_{T1} = P_{T2}/P_{T1} = f(M, \gamma)$$

When the external flow is rarefied, this relation between the measured pressure and the external total pressure breaks down. The shock that forms around the probe becomes diffuse due to a reduced number of molecular collisions. The pressure loss across the shock

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is diminished, and as a result the measured pressure, P_{PG} is larger than that predicted by continuum theory, P_{T2}.

Flow conditions for which continuum results are invalid can be divided into two regimes: transition and near free molecular. In the transition regime the correction factor P_{PG}/P_{T2} is given by some function g,

$$P_{PG}/P_{T2} = g(M, Re_P, T_T/T_W, \gamma)$$

where Re_P is defined using postshock flow properties and the outer diameter of the probe as a length scale. A first collision extension of free molecular flow theory has been used to model the transition flow with some success.[8] There is, however, no comprehensive theory describing the behavior of the probe flow or the functional relationship for this flow regime. There does not appear to have been any previous attempt to compute these flows in detail.

Figure 1 shows experimental data for transition flows taken from Ref. 2 for the correction factor as a function of Re_P and (T_T/T_W). At high Reynolds numbers, corresponding to continuum flows, the correction factor is close to unity. As the Reynolds number drops, the flow becomes transitional and the pressure correction rises. Heated flows, for which $(T_T/T_W) > 1$, show lower correction values. Further experimental results summarized by Fisher[3] have shown that the dependence of the correction on Mach number is small in the transition regime, particularly when a modified Reynolds number is used:

$$Re_M = Re_P \times \sqrt{(\rho_1/\rho_2)}$$

The near free molecular regime occurs at extremely low Reynolds numbers corresponding to highly rarefied external flow conditions. This flow regime is characterized by large pressure corrections that show a significant Mach number dependence. The Reynolds number and temperature dependences are qualitatively the same as for transition flows. In this type of flow the shock that forms in front of the probe is sufficiently diffuse so that a significant number of the gas molecules pass through without undergoing collisions. The pressure measured by the probe depends strongly on what fraction of these collision-free molecules pass directly to the end of the probe and what fraction collide with the probe walls and are thermally accommodated before reaching the end of the probe. A larger number of molecules arriving at the end of the probe without first colliding with the walls will result in a higher pressure as measured by the probe. In flows with high Mach number, the bulk velocity of the gas is much larger than the thermal velocity of individual molecules, and as a result a large fraction of the collision-free molecules reach the probe end without colliding with the walls. In lower Mach number flows, the speed ratio is lower, and a larger fraction of molecules are accommodated before reaching the end of the probe. As a result, in the near free molecular regime the pressure measured by a pitot probe P_{PG} and the correction required P_{PG}/P_{T2} both increase with an increasing Mach number.

As the probe Reynolds number decreases and the shock becomes increasingly diffuse, the pressure measured by the probe and, thus,

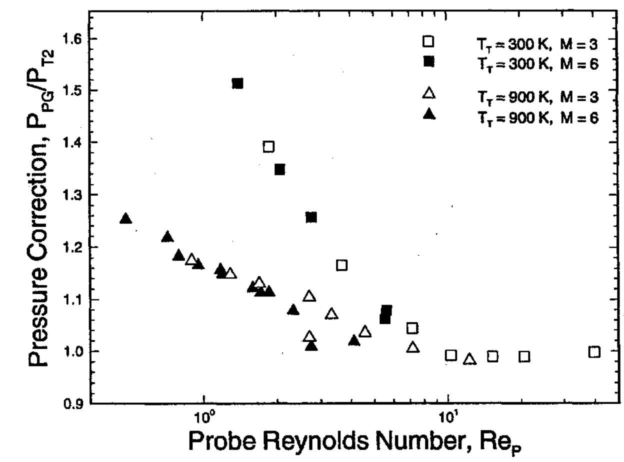

Fig. 1 Experimentally measured pressure correction factors.[2]

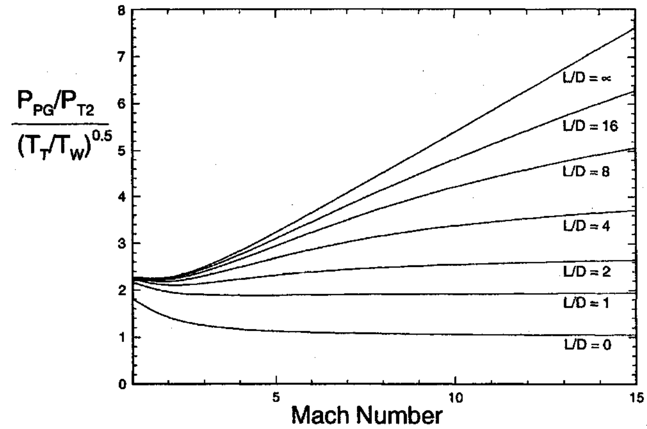

Fig. 2 Predicted free molecular limiting values for pressure correction factors.

the pressure correction should approach asymptotically a limiting value that corresponds to collisionless flow. Hughes and deLeeuw[9] have presented a theoretical model for calculating the pressure measured by a probe under free molecular conditions. This model assumes that there is no interaction between incoming and reflected particles and that particles undergo perfectly diffuse reflection at the probe wall. Figure 2 shows the dependence of the theoretical collisionless correction on Mach number, temperature, and length-to-diameter ratio of the probe. (A similar figure appears in Ref. 10.)

The boundary between the continuum and transition regimes can be defined as the point where the correction factor begins to deviate from unity. As seen in Fig. 1, this point occurs at Reynolds numbers on the order of 10^1, which corresponds to a freestream Knudsen number on the order of 10^{-1}. This appears to be insensitive to the Mach number and temperature conditions of the flow. The boundary between transition and near free molecular flow is not as clear. It can be defined as the point at which the Mach number dependence of the pressure correction factor becomes significant. The Reynolds number at which this occurs is highly dependent on the temperature, Mach number, and length-to-diameter ratio of the pitot probe.

Computational Approach

The numerical simulations performed in this study employ the DSMC method developed originally by Bird.[11] This technique models a gas flow on the molecular level and is, thus, well suited to simulating rarefied or nonequilibrium gas flows for which continuum fluid mechanics are not applicable. Actual gas molecules are simulated using a smaller number of model particles whose paths are traced through the flowfield. Particle motion is decoupled from collisions by considering time steps that are small compared to the local mean collision times. Particle collisions are treated stochastically using results from kinetic theory and include transfer of energy between internal modes. Macroscopic flow quantities (velocity, density, temperatures, etc.) are obtained by sampling particle properties over a large number of time steps once a steady state has been achieved.

The present study employs an efficient DSMC code suitable for use on vector supercomputers or workstations.[12] The pitot probe geometry is modeled by axisymmetric flow into a cylindrical tube with a closed end. Figure 3 shows a schematic of the simulated geometry. The simulation begins some distance upstream of the probe with a uniform inlet condition. Particle collisions are calculated using the variable hard sphere (VHS) gas model.[13] A viscosity exponent of 0.2222 is used with a reference temperature of 273 K. The gas is assumed to be frozen both chemically and vibrationally. The probe walls are assumed to be diffusely reflecting with full thermal accommodation at a constant temperature. The pressure measured by the pitot probe P_{PG} is obtained by direct calculation of the momentum transferred by the particles to the wall at the closed end of the probe.

The extent of the flow domain and the computational requirements depend strongly on the Reynolds number of the simulation in question. For the most rarefied conditions considered ($Re_P = 0.1$)

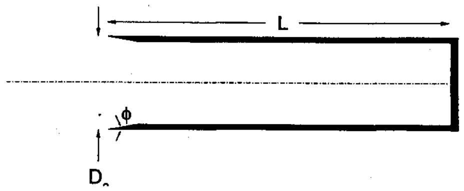

Fig. 3 Schematic of DSMC pitot probe geometry: L/D = 4, chamfer angle ϕ = 10 deg.

the simulation requires about 65,000 particles and a computational grid of 1125 cells, with 45 in the axial direction and 25 in the radial. Time averaging is performed over 5000 time-steps after a transient period of 15,000 time steps. Total execution time is approximately 5 CPU h on an IBM RS6000 workstation or 10 CPU min on a Cray C-90 vector supercomputer. Conditions closest to continuum flow (Re_P = 50) require 300,000 particles and a grid with 4800 cells, 120 axially and 40 radially. Sampling is performed over 5000 time steps after a transient period of 40,000 time steps. In this case, the execution times are approximately 20 CPU h on an RS6000 or 1 CPU h on a C-90. In all simulations, cell sizes are kept within one local mean free path.

Probe Dimensions and Flow Conditions

A standard pitot probe geometry is used for all of the DSMC simulations performed in this study (see Fig. 3). This geometry has an outer diameter of 2.55 cm and an inner diameter of 2.29 cm. The probe entrance has a 10-deg internal chamfer. The length-to-diameter ratio is 4; this value is chosen primarily for computational convenience. In addition, the experimental studies of Fisher[3] indicate little sensitivity of measured pitot pressure to this parameter within the continuum and transition flow regimes. The probe wall temperature is held constant at 300 K.

The inlet flow is uniform, directed toward the probe entrance and parallel to the axis. Three parameters are used to vary the inlet conditions: Mach number, total temperature, and probe Reynolds number. Mach numbers of 3.2 and 6.5 are simulated at two total temperatures, 300 and 900 K. Probe Reynolds number ranges from 0.1 to 50. Most of these values are chosen to match conditions investigated experimentally by Stephenson.[2]

Results

Flowfield Characteristics

To ensure the validity of pressure correction factors computed by DSMC, it is important to verify that the method generates a solution that closely approximates the actual problem. The simulated flowfield must display the characteristics expected in the probe flow, and these features should change in the proper fashion as the flow parameters change. Figures 4–6 show several different types of plots that are used to make this type of qualitative evaluation of simulation results.

Figures 4a and 4b show contour plots of flow properties produced by DSMC. A region in the vicinity of the probe entrance is shown. Figure 4a plots contours of translational temperature (at 100 K intervals) resulting from an inlet total temperature of 900 K and Mach number of 3.2. The upper half of the diagram shows results for a near continuum case (Re_P = 12), whereas the lower half shows a highly rarefied case (Re_P = 0.50). Figure 4b shows (evenly spaced) contours of normalized static pressure resulting from inlet conditions of 300 K total temperature, Mach number of 3.2, and the two previously considered Reynolds numbers. Pressures are normalized using the appropriate ideal postshock total pressure P_{T2}. These figures demonstrate the ability of DSMC to simulate the flowfield characteristics that are expected in the vicinity of the pitot probe. All four cases show a shock forming in front of the probe entrance followed by gradual stagnation and equilibration of the flow as it approaches the back wall of the probe. Comparison of the flows computed at high and low Reynolds number illustrates the marked difference between the continuum and rarefied regimes. The continuum cases show well-formed, thin shocks located close to the probe entrance. The rarefied flow results show very thick, diffuse shocks that extend a significant distance ahead of the probe. This is consistent with the

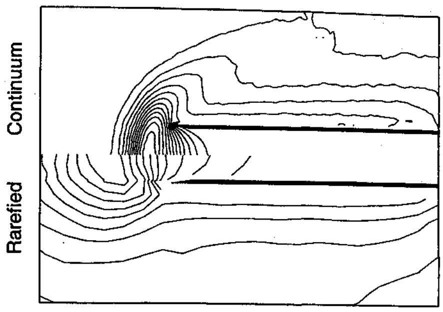

Fig. 4a Comparison of translational temperature contours computed by DSMC for continuum and rarefied flow conditions.

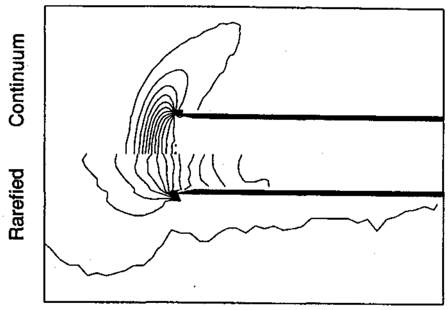

Fig. 4b Comparison of pressure contours computed by DSMC for continuum and rarefied flow conditions.

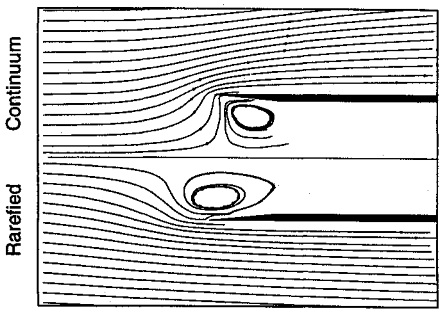

Fig. 5 Comparison of streamlines computed by DSMC for continuum and rarefied flow conditions.

expected behavior: transition flows exhibit weakly formed shocks leading to a reduced pressure loss.

Figure 5 plots streamlines calculated by DSMC for flow conditions of total temperature equal to 300 K and Mach number of 3.2. Again, the upper half of the diagram shows results for a near continuum case (Re_P = 12), whereas the lower half shows a highly rarefied case (Re_P = 0.50). As before, these plots show the effects of decreasing Reynolds number on the shock structure. The continuum case again shows a highly localized shock with the oncoming streamlines being deflected at a sharp angle directly ahead of the probe entrance. The area of circulation that is evident behind the shock is completely within the entrance. The rarefied case shows a diffuse shock; streamlines are deflected much more gradually beginning farther ahead of the probe. The circulation structure now protrudes out from the probe entrance.

A more quantitative evaluation of simulation results can be obtained by considering the behavior of results along the axis of symmetry. Figures 6a and 6b show axial profiles of normalized pressure and translational temperature, respectively, resulting from simulations with T_T = 300 K and a Mach number of 3.2. Three different Reynolds number cases are shown corresponding to a highly continuum flow (Re_P = 50), a transition flow (Re_P = 5), and a near continuum case (Re_P = 12). The probe entrance is located at z = 0. These figures show the features of normal shocks located immediately ahead of the probe. The rarefaction trends already mentioned are seen clearly in the profiles. The shock thickness for the transition flow, defined using the maximum gradient of pressure, is an order of magnitude larger than that of the highest Reynolds number case. The temperature profiles indicate that the rarefied shocks are located

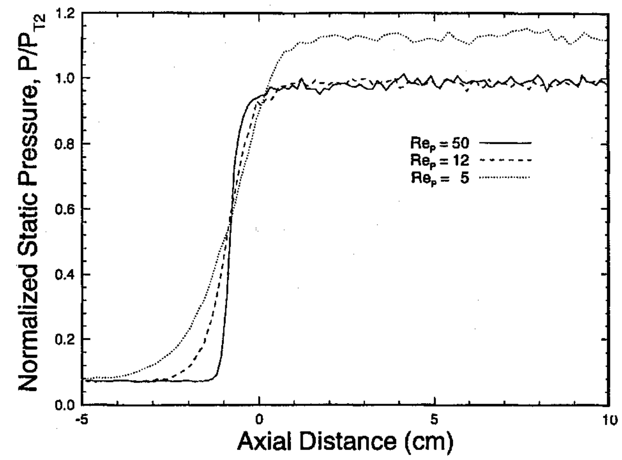

Fig. 6a Profiles of static pressure along probe axis for $M = 3.2$, $T_T = 300$ K flows.

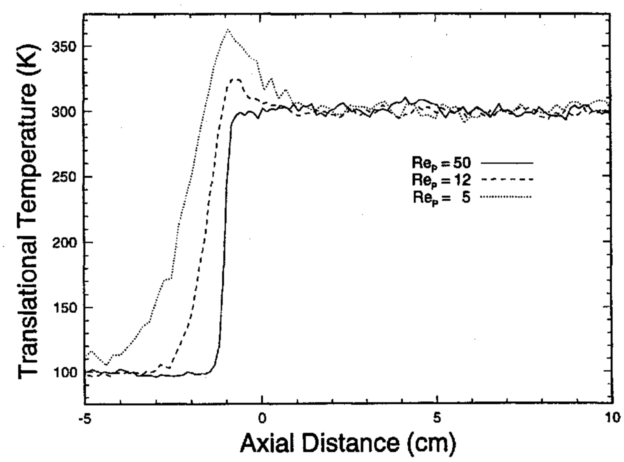

Fig. 6b Profiles of translational temperature along probe axis for $M = 3.2$, $T_T = 300$ K flows.

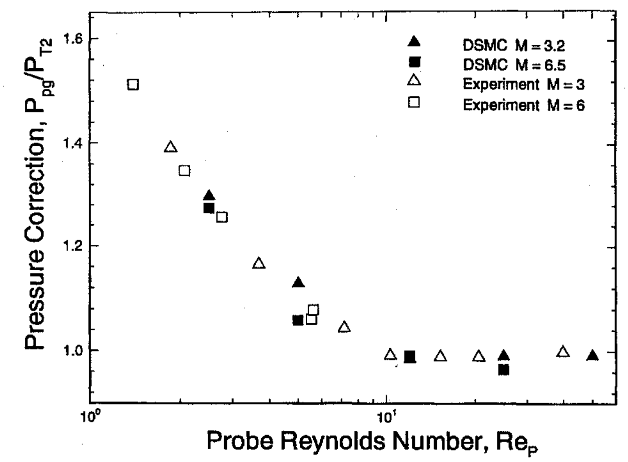

Fig. 7a Comparison of pressure correction factors computed by DSMC with experiment[2] for $T_T = 300$ K flows.

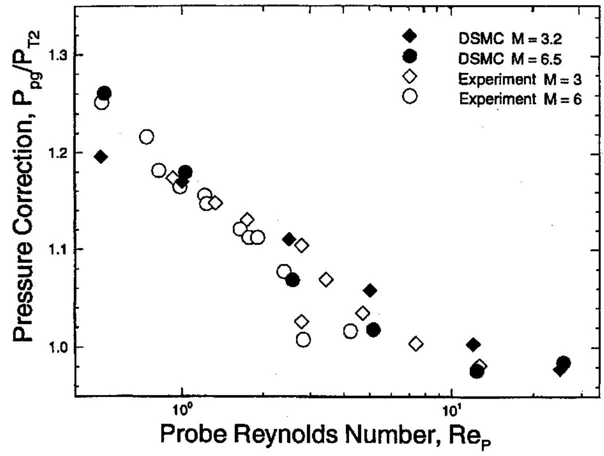

Fig. 7b Comparison of pressure correction factors computed by DSMC with experiment[2] for $T_T = 900$ K flows.

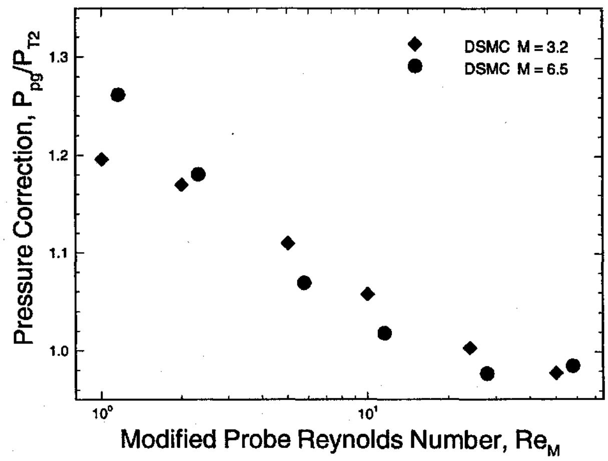

Fig. 7c Pressure correction factors computed by DSMC for $T_T = 900$ K flows plotted vs modified probe Reynolds number.

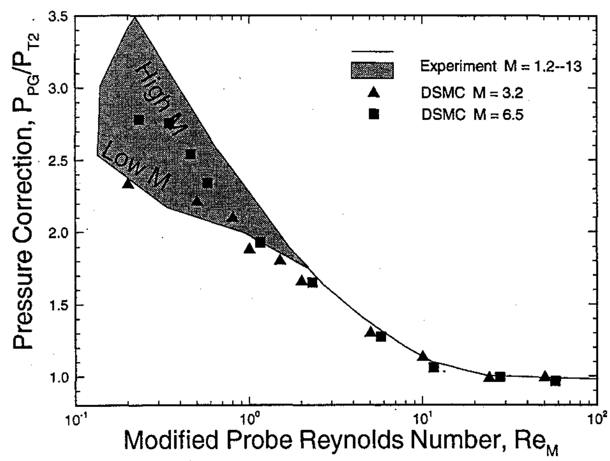

Fig. 7d Comparison of pressure corrections factors computed by DSMC with experiment[3] for $T_T = 300$ K flows.

significantly ahead of the probe entrance. The peak temperatures seen in Fig. 6b are also noteworthy. The two lower Reynolds number cases have maximum temperatures that are significantly higher than the postshock equilibrium temperature, whereas the continuum case equilibrates immediately. The highly peaked translational temperatures are consistent with increasing nonequilibrium effects in the more rarefied flows. Insufficient collisions are present to equilibrate the translational and rotational modes of the gas.

Comparison of Pressure Corrections with Experimental Data

To determine the ability of DSMC to predict accurately the pressure correction factor for a given flow condition, calculated corrections are plotted with experimental results. Figures 7a and 7b show comparisons between DSMC and experimental results from Stephenson[2] for transition and near continuum flow conditions. Figure 7a displays corrections for flows with a total temperature of 300 K, whereas Fig. 7b gives results for 900 K total temperature flows. DSMC calculations were made at two Mach numbers, 3.2 and 6.5, which are comparable to the experimental conditions. Excellent agreement is found between the numerical and experimental correction factors throughout the continuum and transition regimes, particularly for the unheated flows ($T_T = T_W$) shown in Fig. 7a. Behavior of the numerical results near $Re_P = 10$, the boundary between continuum and transition, is noteworthy. Both figures show pressure corrections dropping below 1.0 in this boundary region followed by a rise toward unity; this behavior has been well established by experiments.[3] DSMC results demonstrate this same near continuum behavior.

The DSMC calculations are performed using a probe length-to-diameter ratio that differed greatly from that used by Stephenson (4 as compared to 32). The pressure corrections are independent of this geometric parameter at high Reynolds numbers but develop a strong dependence on L/D in the near free molecular regime. Figure 2 shows the effect of this geometric parameter at the free molecular limit. As a result, extension of the comparisons shown in Figs. 7a and 7b to lower Reynolds numbers is inappropriate because it is difficult to separate the L/D effect from transitional behavior.

Figures 7c and 7d show pressure corrections plotted against modified Reynolds number Re_M. Potter and Bailey[14] have shown the data plotted vs this parameter display minimal dependence on Mach number throughout the transition regime. Figure 7c shows calculated corrections for flows at a total temperature of 900 K and Mach numbers of 3.2 and 6.5. Elimination of Mach number dependence is achieved to some degree, except for the lowest Reynolds number shown, which corresponds to the beginning of the near free molecular regime. Here the expected Mach number dependence is observed with the higher Mach number flow resulting in a higher output pressure due to the more directed nature of the inflow stream and, thus, a larger correction factor. Figure 7d shows a comparison between DSMC calculations and experimental results summarized by Fisher[3] for unheated flows ($T_T = 300$ K). Good agreement is found for continuum and transition conditions. DSMC predicts the Reynolds number corresponding to the onset of transition ($Re_M = 25$). The large amount of scatter in the experimental results at low modified Reynolds number, caused by variations in Mach number and probe dimensions, prevents quantitative comparison of near free molecular results, but the numerical results do demonstrate the correct qualitative behavior in terms of Reynolds number and Mach number dependencies. The onset of significant Mach number dependence in the DSMC results occurs for a value of modified Reynolds number on the order of 10^0, which is reasonably consistent with the experimental data.

Near Free Molecular Behavior

Figure 8a shows the low Reynolds number behavior of DSMC pressure corrections for simulations with the flow total temperature equal to the wall temperature (300 K). The theoretical limiting values, calculated using the Hughes and deLeeuw[9] model, are shown for the two Mach numbers considered. The results display behavior similar to that obtained in low Reynolds number experimental studies[3] deviating from the general upward trend of the transition regime and then approaching asymptotically a limiting value. The asymptotic limits of the two data trends appear to be reasonably consistent with the theoretical model.

Figure 8b shows low Reynolds number behavior for heated flows ($T_T = 900$ K) at two Mach numbers, with the appropriate free molecular limits. The results are reasonably consistent with the expected behavior and theoretical limits. Pressure corrections show the expected effect of Mach number: nearly independent of Mach number throughout the transition regime but developing a strong dependence as the near free molecular regime is traversed. Higher Mach number flows show larger correction factors due to the higher collisionless limits. The two data trends approach the appropriate theoretical limits in a consistent fashion, the rate of increase beginning to fall off as the Reynolds number decreases.

It is uncertain whether the numerical results do approach an asymptotic limit, further simulations at even lower Reynolds numbers would be needed to establish this behavior. In the rarefied extreme, the boundary condition used at the end of the probe does not adequately represent the actual conditions. As a result, probe Reynolds numbers below 0.1 are not considered in the present study. In addition, it is unlikely that one would use a pitot probe to make measurements in this rarefied extreme.

Figure 8c shows the DSMC results for unheated flows ($T_T = 300$ K) at a Mach number of 3.2 plotted vs freestream Knudsen number. The plot shows that the onset of transition occurs at a Knudsen

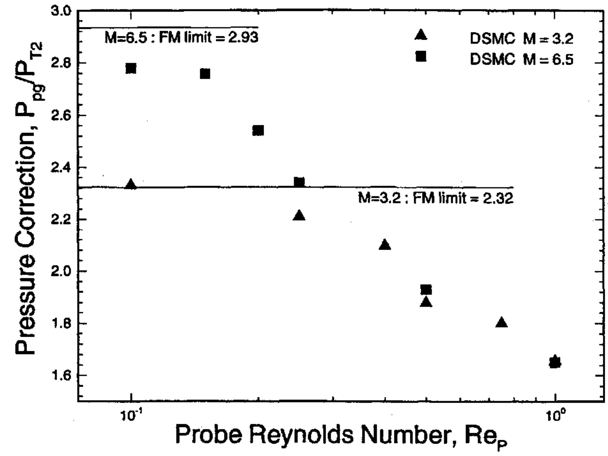

Fig. 8a Comparison of pressure correction factors computed by DSMC with the predicted free molecular limit for $T_T = 300$ K flows.

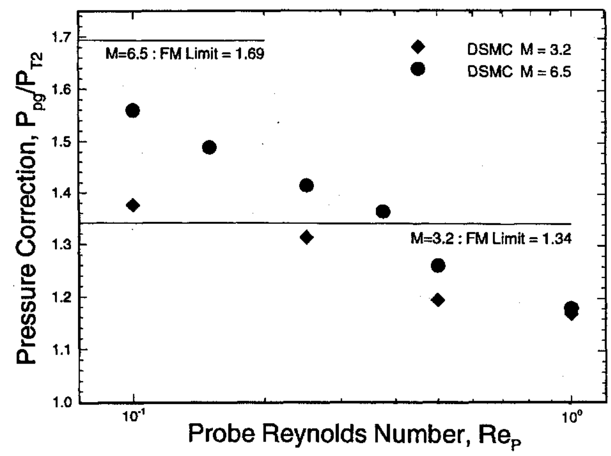

Fig. 8b Comparison of pressure correction factors computed by DSMC with predicted free molecular limits for $T_T = 900$ K flows.

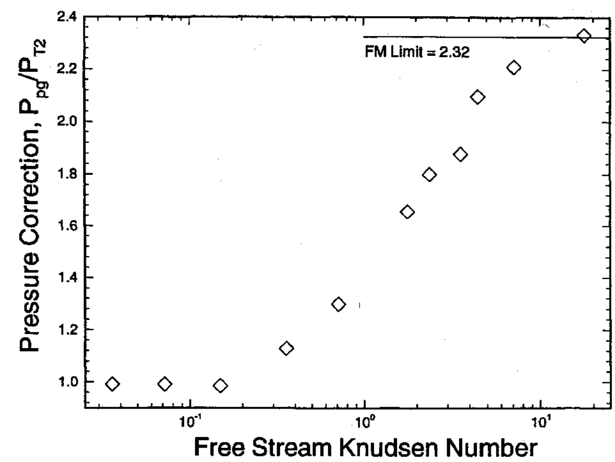

Fig. 8c Pressure correction factors computed by DSMC for $T_T = 300$ K flows plotted vs freestream Knudsen number.

number of about 10^{-1}, which is consistent with the limit of validity of continuum fluid mechanics.[11] The qualitative behavior of the results is consistent with prior experimental data presented by Rogers et al.[8] The two order of magnitude difference in Knudsen number between continuum and free molecular conditions is also consistent with the findings of Rogers.

Concluding Remarks

The strong agreement obtained between pitot probe correction factors calculated numerically and measured experimentally indicates that DSMC is capable of generating these corrections for transition flows with a high degree of accuracy. Consistency with free molecular and continuum limits determined theoretically for these flows further indicates the accuracy of DSMC calculations for probe conditions across all flow regimes. This is a significant finding as no general theory exists to obtain these predictions and experimental measurements are available only for a limited number of flow conditions and test gases.

Based on the successful results obtained in this investigation, it is proposed that the DSMC method may be used to provide pressure corrections for flow conditions for which experimental data are not available. For example, in the resistojet studies described in Refs. 4 and 5, a total temperature of 700 K required corrections to be estimated from the data available from the measurements of Stephenson obtained at values of 500 and 900 K. The ability to obtain such data numerically should result in improved accuracy for measurements made with pitot probes. A further consequence of the satisfactory results obtained for DSMC simulations of nitrogen probe flows is that numerical simulations may be extended to other gas species of relevance to propulsion concerns, such as argon and hydrogen.

Acknowledgments

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References

[1]Liepmann, H. W., and Roshko, A., *Elements of Gasdynamics*, Wiley, New York, 1957, p. 149.

[2]Stephenson, W. B., "Use of the Pitot Tube in Very Low Density Flows," Arnold Engineering Development Center, AEDC-TR-81-11, Arnold AFB, TN, Oct. 1981.

[3]Fisher, S. S., "The Effect of Rarefaction on Impact Pressure Measurements in Supersonic Flows," *Rarefied Gas Dynamics*, Vol. 1, Plenum, New York, 1985, pp. 461–468.

[4]Boyd, I. D., Penko, P. F., Meissner, D. L., and DeWitt, K. J., "Experimental and Numerical Investigations of Low-Density Nozzle and Plume Flows of Nitrogen," *AIAA Journal*, Vol. 30, No. 10, 1992, pp. 2453–2461.

[5]Penko, P. F., Boyd, I. D., Meissner, D. L., and DeWitt, K. J., "Analysis and Measurement of a Small Nozzle Plume in Vacuum," *Journal of Propulsion and Power*, Vol. 9, No. 4, 1993, pp. 646–648.

[6]Boyd, I. D., Beattie, D. R., and Cappelli, M. A., "Numerical and Experimental Investigations of Low-Density Supersonic Jets of Hydrogen," *Journal of Fluid Mechanics*, Vol. 280, 1994, pp. 41–67.

[7]Boyd, I. D., Jafry, Y., and Vanden Beukel, J., "Particle Simulation of Helium Microthruster Flows," *Journal of Spacecraft and Rockets*, Vol. 31, No. 2, 1994, pp. 271–277.

[8]Rogers, K. W., Wainwright, J. B., and Touryan, K. J., "Impact and Static Pressure Measurements in High Speed Flows with Transitional Knudsen Numbers," *Rarefied Gas Dynamics*, Vol. 2, Academic, New York, 1966, pp. 151–174.

[9]Hughes, P. C., and deLeeuw, J. H., "Theory for the Free Molecule Impact Probe at an Angle of Attack," *Rarefied Gas Dynamics*, Vol. 1, Academic, New York, 1965, pp. 653–676.

[10]Fisher, S. S., Yurkanin, D. J., Graybeal, A., and Scott, J. E., Jr.,"Measurements of the Impact Probe Rarefaction Effect in High Mach Number $\gamma = 7/5$ CO_2 Flows," *Rarefied Gas Dynamics*, Vol. 1, Univ. of Tokyo Press, Tokyo, Japan, 1984, pp. 405–413.

[11]Bird, G. A., *Molecular Gas Dynamics and the Direct Simulation of Gas Flows*, Oxford Univ. Press, New York, 1994, pp. 18, 19.

[12]Boyd, I. D., "Vectorization of a Monte Carlo Simulation Scheme for Nonequilibrium Gas Dynamics," *Journal of Computational Physics*, Vol. 96, No. 2, 1991, pp. 411–427.

[13]Bird, G. A., "Monte Carlo Simulation in an Engineering Context," *Rarefied Gas Dynamics*, edited by S. S. Fisher, Vol. 74, Pt. 1, Progress in Astronautics and Aeronautics, AIAA, New York, 1981, pp. 239–255.

[14]Potter, J. L., and Bailey, A. B., "Pressures in the Stagnation Regions of Blunt Bodies in the Viscous-Layer to Merged-Layer Regimes of Rarefied Flow," Arnold Engineering Development Center, AEDC-TDR-63-168, Arnold AFB, TN, Sept. 1963.